Some remarks on Truth and Logic

Gabriel Sandu, Department of philosophy, University of Helsinki

Department of Philosophy, Institute of Logic of the Czech Academy of Sciences

Some of the main points of the talk

- One often forgets that logical formalisms are just "useful fictions"

- Formal logic often leads to a "desemanticalization" of our ordinary notions

- Semantical content is lost

- I will use "is true" as a case study.

Frege (1879) on "Is a fact"

We can imagine a language in which the proposition "Archimedes perished at the capture of Syracuse" would be expressible thus: "The violent death of Archimedes at the capture of Syracuse is a fact". To be sure one can distinguish between subject and predicate here, too, if one wishes to do so, but the the subject contains the whole content, and the predicate serves only to turn the whole content into a judgement. Such a language would have only a single predicate for all judgements, namely "is a fact". We see that there cannot be any suggestion here of subject and predicate in the ordinary sense. Our ideography is a language of this sort, and in it the sign ⊢ is the common predicate for all judgements (Frege, *Begriffsschrift*,12-13)

How "is true" is used: Strawson (1950)

In many of the cases in which we are doing something besides merely stating that X is Y, we have available, for use in suitable contexts, certain abbreviatory devices which enable us to state that X is Y ....without using the sentence pattern 'X is Y'. ....We may state that X is Y by saying 'It is indeed' or 'That's true'. (Strawson 1950; 174-175)

First conclusion (Frapolli 2012):

- "is true" is a contentless expression

- It is intimately linked to nominalization

- It is used for endorsing contents which have been asserted in a different context

- It is used for stressing that a paricular content can be used safely

How truth is used: Adverbs

- IIlocutionary adverbs:

  seriously, confidentially, truthfully, frankly

- Evaluative adverbs:

  sadly, fortunately, correctly

- Evidential adverbs:

  Allegedly, obviously, trully

- Evaluative and evidential adverbs indicate a contextual property of propositions

- They do not contribute to the truth-conditions of the utterance in which they occur

Carston 2002: adverbs

It seems that the propositional form (and hence the truth-conditional content) of these utterances does not include the contribution made by the adverbial. Where then, do these elements make their contribution? ....There is a neat answer to this in the system Sperber and Wilson have developped: they contribute to a higher-level explicature (Carston 2002: 121)

Pragmatical considerations: Classification of truth-ascriptions

- Singular truth-ascriptions:

A1   Exhibitive: "Snow is white" is true

A2   Blind: His claim was true

- General truth-ascriptions:

B1   Exhibitive: The theory of evolution is a true theory

B2   Blind: Everything the Pope says is true

Second conclusion

- "truly" is evidential: it makes its contribution to a second-order explicature.

- It suggests the presence of a further speech act in which the speaker endorses the proposition expressed.

- "She spoke truly" is equivalent with "What she said is true" and "She told the truth"

- "is true" makes a contribution to a second-order explicature

Ramsey (1927): truth-bearers

- Three classes of entities to which "True" and "false" are applied:

1. Mental states: beliefs, judgements, opinions, conjectures

2. Statements of indicative sentences

3. Propositions: the objects of judgement and the meaning of sentences

- "Her belief is true" has the form: "it is a belief that p, and p"

- "John spoke the truth" has the form: "John said that p and p"

where p is a propositional variable.

Ramsey's prosentential theory (Frapolli 2012)

- "Her belief is true" can express various con- tents, depending on the context

- It can express the contents:

  that snow is white

  that China is a big country

  that Helsinki is the capital of Finland, etc.

- Its logical form is "it is a belief that p, and p" where p is a propositional variable.

Third conclusion

- Truth-bearers are contents: propositions, claims, believes, etc

- The contextuality of truth ascriptions: they endorse different contents, depending on the context.

- From a semantical point of view, the truth-predicate is a propositional variable

Ramsey and Tarski: comparison

- Ramsey (1927) focused on: "Her belief is true", "What the Pope says is true", etc.

- He identified propositions as truth-bearers

- Tarski (1933) claimed that

"A true sentence is one which says that the state of affairs is so and so, and the state of affairs is so and so"

Ramsey and Tarski: comparison continued

- Finally Tarski settled for:

(T) x is a true iff p

where 'x' stands for the structural descriptive name of the sentence p.

  - The truth-bearers have changed from content to sentences.

Barwise and Etchmendy (1987)

Truth, as we ordinarily understand the notion, is a property of things like claims, testimony, assertions, beliefs, statements, or propositions. It is not a property of sentences. But the decision to use sentences as the bearers of truth has been proven to be a useful fiction, a good way of getting a certain amount of logic done without bogging down in extralogical questions about the nature of the bearers of truth. But the fiction is harmless only in cases where we can unambiguously associate a claim about the world with each sentence…(Barwise and Etchmendy, 10)

Tarski: the derivation of the Liar

- We stipulate that 'c' names the sentence

  c is false

- From the (T)-skeema we get:

(2)   'c is false' is true iff c is false

- (2) together with the stipulation 'c is false'
  entails

(3) c is true iff c is false

- From (3) and logic we get:

(4) c is true and c is false

Tarski's diagnosis (1933/1983)

- Tarski blames the universality of language:

  In section 1 (Tarski 1933) colloquial
  language is the object of our discus-
  sion. The final conclusion is totally
  negative: In that language it seems to
  be impossible to define the notion of
  truth or even to use this notion in a
  consistent manner and in agreement with
  the laws of logic. (Tarski, p. 153)

Fourth conclusion

- Tarski was not interested in truth in natural (colloquial) language

- He focused on a limited role of "is true" in natural language

- He substituted sentences for contents as truth bearers.

Parsons (1984), Burge (1979): truth-bearers are propositions

- Parsons reformulates the Liar as

  (c) c expresses a false proposition

- Parsons replaces Tarski's (T) schema by the weaker

(6) $\forall x (x$ is a proposition and 'p' expresses x, then x is true iff p)

- Together with the bivalence of propositions, (6) entails

(7) $\forall x (x$ is a proposition and 'p' expresses x, then x is false iff $\neg p$)

If the Liar expresses a proposition, it is a true one

- Suppose that c expresses a proposition x. Then by (6):

(8) x is true iff c expresses a false proposition

- Suppose x is not true. By exist. gen.

(9) $\exists x(x$ is a proposition $\wedge \neg(x$ is true$) \wedge c$ expresses $x)$

- That is, c expresses a false proposition

- But then by (8), x is true. Thus:

(10) $\forall x(x$ is a proposition and $'c'$ expresses $x \rightarrow x$ is true$)$

The Liar does not express a proposition

- Recall

(10) $\forall x(x$ is a proposition and 'c' expresses x $\rightarrow x$ is true)

- (10) is equivalent to

(11) $\neg\exists x(x$ is a proposition $\wedge\neg(x$ is true) $\wedge\ c$ expresses $x)$

- But then there is no proposition that c expresses

- For if c expressed y, then by (10) y must be true. By (8) c expresses a false proposition.

- But this contradicts (11)

Read: saying something

- 'x : p' with the intended meaning: x says that p.

- Read replaces Tarski's (T) schema with:

(A) $$Tr(x) \leftrightarrow \forall p(x : p \rightarrow p)$$

where x stands for sentences.

- That is, x is true iff for any proposition p, if x says that p, then p.

- Read is aware that (A) makes true all sentences which says nothing.

- Constraint: (A) is to be applied only to sentences which say something.

Read's conclusion

- All the instances of the (A)- schema are
  true.

- The liar sentence c turns out to be false
  without contradiction,

- The laws of classical logic still hold.

Parson: sentences as truth-bearers

- Parson defines a truth-predicate for sentences:

(*) $Tr('p') \leftrightarrow \exists x(x$ is a proposition $\wedge$ (x is true) $\wedge$ ($'p'$ expresses x))

- (*) and (6) imply:

(T*) $\exists x(x$ is a proposition $\wedge$ ($'p'$ expresses x))$\rightarrow (Tr('p') \leftrightarrow p)$

- Analogously, for a falsity-predicate:

(F*) $\exists x(x$ is a proposition $\wedge$ ($'p'$ expresses x))$\rightarrow (F('p') \leftrightarrow \neg p)$

Parsons and Read

- Parson's T-schema for sentences:

(25) $Tr('p') \leftrightarrow \exists x (x$ is a proposition $\wedge$ $('p'$ expresses $x) \wedge x)$

- A sentence is true iff it expresses a proposition which is the case

- Read's T-schema for sentences (changing notation):

(A) $Tr('p') \leftrightarrow \forall x('p'$ says that $x \rightarrow x)$

- A sentence is true iff everything which it says is the case.

Is there a Liar paradox? (Frapolli 2012)

- A chain of truth or falsity ascriptions do not express a content unless there is a level in which a genuine assertion has been made.

- Liar sentences like

1. I am lying, and

2. This sentence is false

do not express a truth-evaluable content.

- The act in which the Liar is uttered is ungrounded.

- The Liar paradox arises out of an incorrect use of truth.

The usefulness of the Liar

- Self-reflexibility is not the source of the trouble

- The Liar does not refers to a context from which the content can be extracted; nor does it permit such a context to be provided.

- The Liar does not prove the inconsistency of the notion of truth

- Instead it shows the domain within which the notion can be applied.

Deflationism approaches to truth

- Truth does not have a nature which can be discovered by a sophisticated philosophical analysis

- Quine's disquotationalism: the truth predicate cancells the quotation marks names

- Horwich: everything one needs to know about truth is exhausted by the infinite number of instances of the schema :

(Min) The proposition that p is true iff p.

An inferentialism approach to truth

- "is true" is treated like a logical constant

- Inferential approach to logical constants: their meaning is given by introduction and elimination rules, e.g.

$$\frac{A \quad B}{A \wedge B} \quad \frac{A \wedge B}{A} \quad \frac{A \wedge B}{B}$$

Constraints on Introduction and Elimination rules: Dummett and Pravitz

- The rules are structural (schematic): Any propositional symbol can replace A and B

- Dummett (1991), global harmony (conservativity): adding a new logical constant produces a conservative extension

- Dummett (1991), local harmony (reduction).

Truth and inferentialism

- Introduction and elimination rules

- Truth in

$$\frac{A}{Tr(a)}$$

- Truth out

$$\frac{Tr(a)}{A}$$

where A is a sentence of the object language and "a" is a term or a name of the sentence.

Truth and semantical information (Hodes 2004)

- The rules

$$\frac{\text{Snow is white}}{Tr(a)}$$

and

$$\frac{Tr(a)}{\text{Snow is white}}$$

require semantical information that "a" is the name of "Snow is white".

- The rules are not purely structural: they rely on a nominalization process

Conclusion

- " is true" is not a purely logical constant: it forces us to make a detour through semantics

- The rules *Truth In* and *Truth out* violate global harmony (conservativity) and structurality

- The source of the nonconservativity is nominalization

- "is true" satisfies only local harmony (reduction)

- It is impossible to reconcile deflationist truth theories with inferentialism

- A complete account of truth must have two parts:

1. nominalization rules, and

2. inference rules

- One cannot define truth without using semantical notions

Suggestion for a solution: A "quasi-logical" or syntactical nominalization

- Require that names in *Truth In* and *Truth out* be canonical names.

- Tarski and Quine developed a theory of canonical names from the 30's

- Quine's inverted commas: names without a structure which denote the relevant sentence

- Their semantic behaviour is similar to that of ordinary names

- Criticisms by Davidson

Tarski's structural descriptions

- It uses the concatenation operator to obtain structural descriptions

- In both structural descriptions and canonical names, the names are formed independently of the semantical content of the expression.

The content of canonical names (Rivenc 2004, Boyer forthcoming)

- The rules
$$\frac{\text{Snow is white}}{Tr(\ll \text{Snow is white} \gg)}$$
  and
$$\frac{Tr(\ll \text{Snow is white} \gg)}{\text{Snow is white}}$$

- The content of $\ll$ Snow is white $\gg$ must be active

- This must be so if the T-equivalences are analytical

- The target of a canonical name is an active semantic content, not a syntactical expression

- The expression in the quotation marks or the canonical name is not only mentioned but also used, although in a deviant way

Further conclusions on canonical names

- The requirement that T-sentences be formulated in the speaker's idiolect (Field, Horwich) is another facet of the expression in quotation marks being semantically active.

- Nominalization is not only a syntactical operation

- With Quine's and Tarski's structural descriptions (inverted commas, quotation marks), the semantic content is lost.

Truth as an operator

- Truth In
$$\frac{\text{Snow is white}}{Tr(\text{Snow is white})}$$

- Truth Out
$$\frac{Tr(\text{Snow is white})}{\text{Snow is white}}$$

- The resulting system is conservative over logic: global harmony, local harmony, and purely structurality

- But is is useless in expressive power (just because it lacks nominalization)

- It does not lead to semantic ascent

- It has lost semantical content

Some tentative conclusions

- Formal languages and formalisms are cognitive artifacts (Catarina Dutilh Novaes, forthcoming)

- Formalisms and formalization have the effect of *desemantification*: decoupling of symbols from their usual meaningful function.

- It allows for sensori-motor manipulation of the symbols

- Barwise and Etchmendy: formal languages are useful fictions.