Logic and Values

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Contents

What is logic?

Standard view:

Logic is the science of a **correct reasoning** and valid argument.

Aristotle:

"We must first state what our inquiry is about and what its object is, saying that it is about **demonstration** and that its object is demonstrative science".

Kleene:

"Logic has the important function of saying what **follows** from what".

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Foundational questions:

- What is reasoning? How should it be represented for a logical reconstruction?
  - Should it be treated as a **process**?
  - What **kind** of process?
  - What is the **bearer** of this process?
- On what **grounds** can we qualify reasoning as correct or incorrect?
- What is the **basis** for **logical rules**?

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Some foundational strategies:

The psychologistic approach

Logical rules essentially reflect the process of sound human thinking; they are based upon the so-called “laws of thought” and prescribe how we should think.

The linguistic approach

Logical rules are rules for operating with language expressions. They represent regularities corresponding to structural features of the given linguistic system.

The transcendentalistic approach

Logical rules represent fundamental *a priori* structures of consciousness by means of which we synthesize our concepts and intuitions to acquire knowledge of the world as it is given in the process of apperception.

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- They all interlink logic with an **activity** of some agent, whether it be the cognitive or linguistic activity (practice) of a human being or the conscious activity of a transcendental subject in the Kantian sense.
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An alleged normativity of logic

Based on such an understanding of the subject-matter of logic one arrives at a conception of logic as a **normative** discipline.

According to this conception, logic is considered to be a discipline which provides **norms** for thought or reasoning.

Peirce: Logic is the theory of self-controlled, or deliberate, thought. Logic and the other normative sciences ask not what **is** but what **ought to be**.

Critical Thinking Web:

Studying the correct principles of reasoning is not the same as studying the **psychology** of reasoning. Logic is the former discipline, and it tells us how we **ought to** reason if we want to reason correctly.

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Peirce's idea of Normative Science

Classification of Sciences:

All sciences are either,
A. Science of Discovery; B. Science of Review; or C. Practical Science.

Science of Discovery is either,
I. Mathematics; II. Philosophy; or III. Idioscopy (special sciences).

Philosophy is devided into
a. Phenomenology; *b*. Normative Science; *c*. Metaphysics.

Normative Science has three divisions:
i. Esthetics; ii. Ethics; iii. Logic.

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Peirce's characterization of a normative science

- The study of **what ought to be**, of norms or rules which need not but ought to be followed.
- The "ought" implies ideals, ends, purposes which attract and guide deliberate **conduct** (behavior);
- The science of **ideals**, or of that which is admirable.
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Norms and values

Norms:

- binding principles of **right** actions serving to guide, control, or regulate **proper** and **acceptable** behavior;
- **behavioral rules** that is considered valid.

Values:

- “right”, “proper”, “acceptable”, “desirable”, “admirable”, etc. are **value concepts**;
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Normative treatment of logic:

- Logical rules represent **logical norms**.
- Logical norms regulate **logical behavior** (logical activity).
- Logical behavior is carried out in accordance with **logical values**.
- Logical values separate **logical goodness** from **logical badness**.

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Normative science and normative systems

The main task of a normative science could be:

- to investigate **existing** normative systems;
- to design (establish) **new** normative systems.

What should be the task of logic as a normative science?

- If this task is to investigate the existing normative systems, then it is an empirical science, and the problem remains what these systems are based upon.
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Problems with the normative treatment of logic:

Foundational Shift

- from the **nature** of logic to (some of) its possible **applications**;
- from the **basic** aspects of a logic to its **design** aspects.

Anthropologization of Logic

- "anthropological treatment" of the subject-mater;
- "anthropological treatment" of the logical laws;
- "anthropological treatment" of the logical values.

Imagine that the mankind will cease to exist.

Would the laws of logic continue then to hold?

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An analogy: mathematics

- Mathematics is the theory of self-controlled, or deliberate, **calculations**.
- Mathematics is the science of **correct** calculations.
- Mathematics is a discipline which provides **norms** for calculations.
- Mathematics tells us how we **ought to** calculate if we want to calculate correctly.
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Foundation of logic: an ontological alternative

“Anthropologic” (anti-realistic) solutions:

- Psychologistic strategy turns logic into a branch of psychology.
- Linguistic strategy relativizes logic with respect to the given languages (linguistic frameworks).
- Transcendentalistic strategy is hardly compatible with the fact of the existence of many (non-classical) logical systems.

An alternative:

If we seek to construe logic as a fully **objective discipline**, it could be reasonable to take a look at an **ontological** (realistic), strategy of justifying logical rules.

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The logical idea of truth values

Gottlob Frege:

"A sentence proper is a proper name, and its *denotation*, if it has one, is a truth-value: the True or the False."

"A statement contains no empty place, and therefore we must take its *denotation* as an object. But this *denotation* is a truth-value. Thus the two truth-values are objects."

Nuel Belnap:

"Truth values were put in play by Frege to be the *denotations* of sentences, in contrast with their *senses*. If I may use '*T*' and '*F*' as names of the two classical truth values, then the story is that the denotation of 'snow is white' is *T* or *F* according as snow is or is not white. What a happy idea!"

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Truth values can be characterized as:

- primitive abstract objects that may serve as denotations for sentences in natural and formal languages,
- abstract entities hypostatized as the equivalence classes of sentences,
- values indicating the degree of truth of sentences,
- entities that can be used to explain the vagueness of concepts,
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The science of truth values

Gottlob Frege:

"Logic is the science of the most general laws of being true."

Jan Łukasiewicz:

"All true propositions denote one and the same object, namely truth, and all false propositions denote one and the same object, namely falsehood. I consider truth and falsehood to be *singular* objects. . . . Ontologically, truth has its analogue in being, and falsehood, in non-being. The objects denoted by propositions are called *logical values*. Truth is the positive, and falsehood is the negative logical value. . . . Logic is the science of objects of a special kind, namely a science of *logical values*."

Logical entities

Logic is the science of **logical entities**.

Let us apply to the World of Logic the famous Fregean distinction between **functions** and **objects**.

Fundamental logical entities:

- logical objects – truth values;
- logical functions – the functions ranged over truth values.

Logical systems are based on logical entities which constitute **logical structures**.

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Logical structures

Propositional **valuational system** $\langle \mathcal{V}, \mathcal{D}, \mathcal{F} \rangle$:

- $\mathcal{V}$ is a non-empty set (of truth values) with at least two elements,
- $\mathcal{D}$ is a non-empty proper (designated) subset of $\mathcal{V}$,
- $\mathcal{F} = \{f_1, \ldots f_n\}$ is a set of (truth-value) functions on $\mathcal{V}$.

Truth-value lattice $(\mathcal{V}, \leq_1 \ldots \leq_n)$:

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- No science is normative with respect to its basic aspects.
- Any science is more or less normative with respect to its design aspects.
- The idea of logic as a normative science is a typical misconception when basic aspects are confused with design aspects.

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